Stats 1 - May / June 2013

(1) a) i) From calc: $\bar{x} = 62.25$ $\quad \Sigma x = 747$

$s = 17.51946...$

ii) $\boxed{\text{MEAN}}$ $\frac{5}{9}(\bar{x} - 32)$

$= \frac{5}{9}(62.25 - 32) = 16.8055...$

$\boxed{\text{SD}} = \frac{5}{9}(s)$

$= \frac{5}{9} \times 17.51946... = 9.733...$

b) $r = 0.997$

The value of r is not affected by linear scaling.

(2) $X \sim N(421, 2.5^2)$

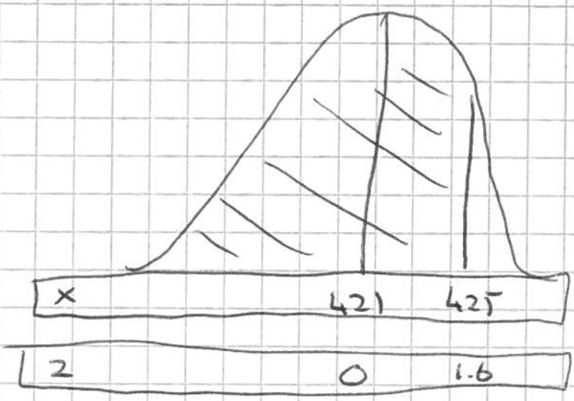
a) i) $P(X = 421) = 0$

ii) $P(X < 425)$

$= P\left(Z < \frac{425 - 421}{2.5}\right)$

$= P(Z < 1.6)$

$= 0.94520$



iii) $P(418 < X < 424)$

$= P\left(\frac{418-421}{2.5} < Z < \frac{424-421}{2.5}\right)$

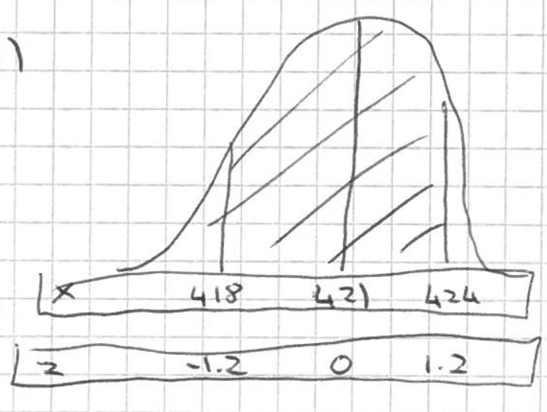
$= P(-1.2 < Z < 1.2)$

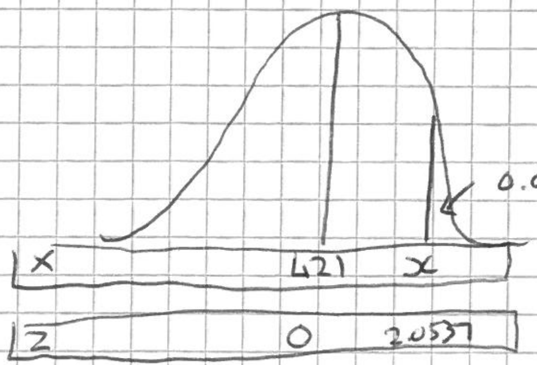
$= P(Z < 1.2) - P(Z < -1.2)$

$= P(Z < 1.2) - [1 - P(Z < 1.2)]$

$= 0.88493 - [1 - 0.88493]$

$= 0.76986$





z value for 0.98 = 2.053

Standardise:

$$\frac{x - 421}{2.5} = 2.0537$$

$$x = 2.5 \times 2.0537 + 421$$

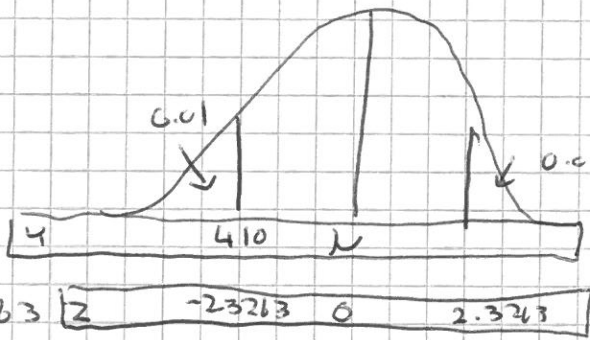
$$= 426.13425$$

c) $Y \sim N(\mu, 3^2)$

$P(Y < 410) = 0.01$

Look up z of 0.99

$= 2.3263$



$\therefore$ z value we need $= -2.3263$

Standardise: $\frac{410 - \mu}{3} = -2.3263$

$$410 = -2.3263 \times 3 + \mu$$

$$\mu = 410 + 2.3263 \times 3 = 416.9708$$

(3) a) i) $X \sim B(40, 0.15)$

$P(X \leq 10) = 0.9701$

ii) $X \sim B(40, 0.5)$

$P(X \geq 25) = 1 - P(X \leq 24)$

$= 1 - 0.9231 = 0.0769$

iii) $X \sim B(40, 0.175)$

$P(X = 2) = {}^{40}C_2 \times 0.175^2 \times 0.825^{38}$

$= 0.0160\ldots$

iv) $X \sim B(40, 0.35)$ [0.85 − 0.5]

$P(10 < X < 15)$

can be: 11, 12, 13, 14

$\therefore P(X \leq 14) - P(X \leq 10)$

$= 0.5721 - 0.1215 = 0.4506$

b) Prob of 1 $= 0.85 - 0.175 = 0.675$

$\therefore$ Mean $= np = 40 \times 0.675 = 27$

(4) a) i) $r_{gy} = \dfrac{S_{gy}}{\sqrt{S_{gg} \times S_{yy}}} = \dfrac{24.15}{\sqrt{0.1196 \times 5880}}$

$= 0.91067\ldots$

ii) $r_{ly} = \dfrac{S_{ly}}{\sqrt{S_{ll} \times S_{yy}}} = \dfrac{10.25}{\sqrt{0.0436 \times 5880}}$

$= 0.64017$

b) $r_{gy} = 0.91 \rightarrow$ strong positive linear correlation between girth + weight

$r_{ly} = 0.64 \rightarrow$ moderate positive linear correlation between length + weight

c) i) $r_{xy} = \dfrac{S_{xy}}{\sqrt{S_{xx} \times S_{yy}}} = \dfrac{5662.97}{\sqrt{5656.15 \times 5880}}$

$= 0.98196$

Y is most strongly correlated with x

ii) $g = 1.25$, $l = 1.15$

$\rightarrow x = 69.3 \times 1.25^2 \times 1.15$

$= 124.52$

iii) $b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{5662.97}{5656.15} = 1.00121$

$a = \bar{y} - b\bar{x} = 116 - 1.00121 \times 115.4$

$= 0.46085$

$\rightarrow y = 0.46085 + 1.00121x$

iv) $r_{xy} = 0.98196 \rightarrow$ very strong positive linear correlation

$b = 1.00121 \rightarrow$ very close to 1

$a = 0.46085 \rightarrow$ close to 0

$\therefore$ Estimate is very likely to be accurate

5) a) i) $P(W \cap T) = 0.9 \times 0.95 = 0.855$

ii) $P(W \cap T') = 0.9 \times 0.05 = 0.045$

$P(W' \cap T) = 0.1 \times 0.95 = 0.095$

Total Prob $= 0.14$

b) i) $P(A \cap D) = 0.9 \times 0.8 = 0.72$

ii) $P(A_W \cap D_W \cap A_T \cap D_T)$

$= 0.72 \times 0.95 \times 1 = 0.684$

iii) 0

iv) $P(A'_W \cap D'_W \cap A'_T \cap D'_T)$

$= 0.1 \times 0.85 \times 0.05$

$= 0.00425$

6) a) $X \sim N(\mu, 0.4^2)$

$\bar{x} = \frac{497.5}{25} = 19.9$ $\quad s = 0.4 \quad n = 25$

98% z multiplier (2 tailed) $= 2.3263$

$\mu = \bar{x} \pm z \times s/\sqrt{n}$

$\mu = 19.9 \pm 2.3263 \times 0.4/\sqrt{25}$

$= 19.9 \pm 0.186104$

$= (19.714, 20.086)$

ii) 20 kg lies within our confidence interval

$\therefore$ no reason to doubt claim

iii) Weight of sand is known to be normally distributed.

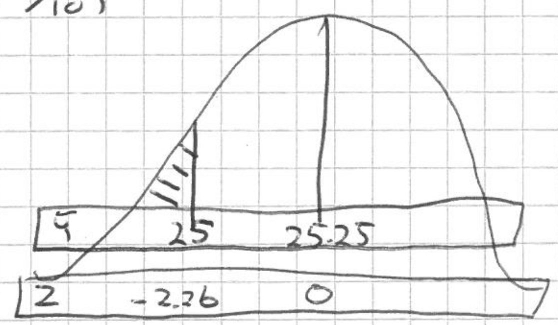
b) $Y \sim N(25.25, 0.35^2)$

$\bar{Y} \sim N(25.25, 0.35^2/10)$

$P(\bar{Y} < 25)$

$= P\left(Z < \frac{25 - 25.25}{0.35/\sqrt{10}}\right)$

$= P(Z < -2.25877)$



$= P(Z < -2.26)$

$= 1 - P(Z < 2.26)$

$= 1 - 0.98809 = 0.01191$

ii) work out 1 first: $(n=1)$

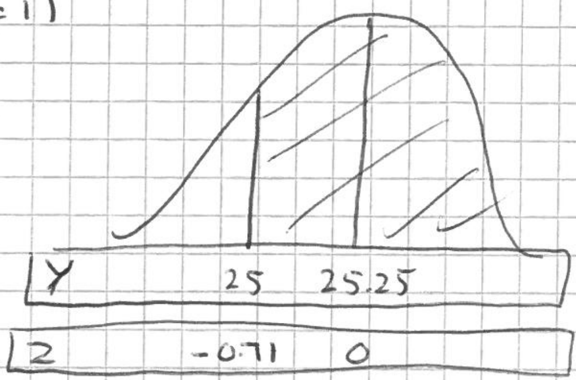
$P(\bar{Y} > 25)$

$= P\left(Z > \frac{25 - 25.25}{0.35}\right)$

$= P(Z > -0.71429)$

$= P(Z < 0.71)$

$= 0.76115$



Now we need all 10

$\rightarrow 0.76115^{10}$

$= 0.06526\ldots$